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MATHEMATICAL MODEL OF STATIC INTERFERENCE IN OPTICAL INTERFERENCE DEVICE

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ABSTRACT

A large number of practical applications of the video surveillance system is based on determining the parameters of vehicle traffic, conducting tests to ensure the safety of the movement of air and sea objects, monitoring the organizations of the interaction of these objects with each other, which is due to the need to track the dynamics of objects. At the same time, an important task is to automatically register, track the relative movement and determine the parameters of the dynamics of objects located in the field of vision of the system. The most difficult, and at the same time urgent task, is tracking spatial multi-tone objects located on a complex dynamic background, the sizes and configurations of which change in the process of tracking them. Particular attention should be paid to organizations monitoring which is conducted in conditions of both low and high external illumination, as well as in conditions where illumination can change over a period of time. This raises many questions related to the analysis of images that cannot be solved without knowledge of the dynamic spatial characteristics of the object or are solved with a loss of quality indicators and time. Hence, there arises an objective need to develop a mathematical model of static interference in an optical interference device.

Key Words: interference, coherence, interferometer, video surveillance.

INTRODUCTION

Currently, there are many optical systems for monitoring the position of objects using interference. The most promising are devices that use the spatial coherence of radiation and are realized in the form of prismatic blocks. Such devices have constant parameters, little dependence on the ambient temperature and other destabilizing factors. A diagram of such an interferometer is shown in Fig. 1.

Figure 1. Diagram of an optical control system for small angular displacements

In this case, there is two-beam interference with the formation of interference fringes. The stripe alternation step will depend on the path difference of the interfering beams. The radiation from the source (laser) illuminates both receiving mirrors of the interferometer. Beam *aa'*, reflected from a deaf mirror, hits a semitransparent mirror, where the beam is reflected into the analysis plane. The second ray *bb'*, passing through a semitransparent mirror, also finds itself in the analysis plane, where an interference pattern is formed.

Let us consider the process of formation of an interference pattern in more detail [1].

Place the base of the interferometer perpendicular to the beam outgoing from the radiation source and falling on some point N of the base plane (Fig. 2). Draw a cross section through the normal ray and base. Then the angle of the opaque mirror in relation to this beam is 45°. Let us denote the distance from the source to the base R, the thickness of the prism *H*, the distance to the analysis plane *l*, the distance between the mirrors *D*. The value of *D* depends on the temperature gradient, and it can be considered constant if we neglect its influence. Let us denote the angle of incidence of the first ray, which is then reflected from the opaque mirror, through φ, and the angle of incidence of the second ray, which then passes through the semitransparent mirror, through ψ. Positive direction for counting angles φ clockwise, angles ψ counterclockwise. We place the origin of the coordinate system at the bottom point of the opaque mirror, direct the Ox axis along the bottom face of the base, and direct the *Oy* axis perpendicular to the *Ox* axis.

If the angles are sufficiently small, the values of the tangents and sines with an accuracy of higher order values can be replaced by the values of the angles measured in radian measure:

$$\sin\varphi=\varphi;\ tg\,\varphi=\varphi.$$

The angles can be considered quite small, for example, when the distance to the source is more than several tens of meters and transverse displacements of several decimeters. The rest of the values in this calculation are also taken with the same accuracy.

The shift of the refractive point of the first ray relative to point *N* is

$$D_1= R\varphi.$$

$$\frac{\varphi}{\varphi'}=n \text{ or } \frac{\varphi}{n}=\varphi',$$

where φ'- is the angle of the refracted ray, n is the refractive index of the prism. The displacement of the refracted ray along the *Ox* axis is

$$H\varphi'=H\varphi/n.$$

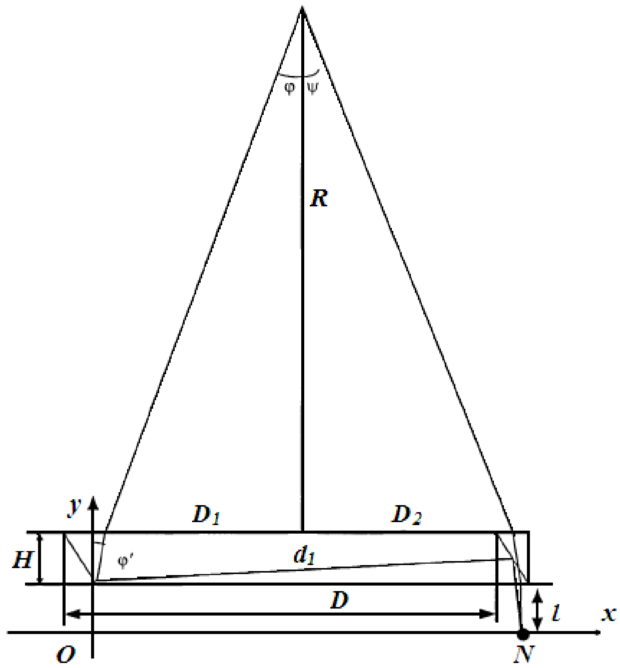

Figure 2. Analysis of the path of interfering rays

According to the law of refraction

According to the law of reflection, the angle of the reflected ray with respect to the *Ox* axis is φ'. Consider a triangle formed by this ray, the *Ox* axis and a semitransparent mirror. Let us denote the value of the segment coinciding with the ray by the symbol *d1*. Then

$$d_1=D(1-\varphi').$$

Consider a triangle formed by a semitransparent mirror, the Ox axis and a ray reflected from a semitransparent mirror. It can be considered an isosceles right-angled triangle with leg φ'D. We neglect the displacement along the *Ox* axis of this ray. Then the geometric course of the incident beam on the semitransparent mirror and the one reflected from it is equal to *D*. Let us denote the angle of inclination of the semitransparent mirror with respect to the transparent one by β. The value of the angle is considered positive if the slope of the angle goes counterclockwise, and negative otherwise. The angle of inclination of the mirror to the *Ox* axis is $45°-\beta$. Then the angle of incidence on the line of exit from the prism of the beam reflected from the semitransparent mirror $\varphi'-2\beta$. The displacement of the beam after exiting the prism along the *Ox* axis is [2,3]

$$S_1=l(\varphi-2n\beta).$$

The coordinate of the point *x*, at which the interfering rays fall on the line of analysis, is

$$x=D(1-\varphi)-l(\varphi-2n\beta)=D+2l\beta n-(D+l)\varphi \quad (1)$$

From formula (1) it can be seen that the position of the representing point on the line of analysis is determined by a linear dependence on the angle φ.

Similarly, for the second ray, we obtain

$$x = R\varphi+ H\varphi/n+ R\psi + H\psi/n +l\psi. \quad (2)$$

Therefore, taking into account formulas (1) and (2), after transformations we obtain

$$(R+H/n + l)(\psi+\varphi)=D(1-\varphi) +2n\beta l. \quad (3)$$

From formula (3) it follows that the position of the representing point on the line of analysis is also determined by the linear dependence on the angle ψ. As a first approximation from formula (3) we obtain

$$\psi + \varphi \approx \frac{D}{(R + H/n + l)} = const. \tag{4}$$

This means that the value of the angle between the two rays arriving at a given point of the line of analysis of the interference pattern, in the first approximation, remains constant for the entire scale of analysis.

The optical path difference between the rays, as follows from the geometric pattern, is [1]

$$\Delta S = \frac{R}{\cos\varphi} - \frac{R}{\cos\psi} + \frac{nH}{\cos\varphi'} - \frac{nH}{\cos\psi'} + \frac{l}{\cos(\varphi - 2\beta)} - \frac{l}{\cos\psi} + nD. \tag{5}$$

Using the expansion in a series $\frac{1}{\cos x} = 1 + \frac{x^2}{2!}$ up to and including the second-order values [7], from formula (5) we obtain

$$\begin{aligned} \Delta S &= R\left(\frac{\varphi^2}{2} - \frac{\psi^2}{2}\right) + nH\left(\frac{\varphi'^2}{2} - \frac{\psi'^2}{2}\right) + l\left(\frac{(\varphi - 2\beta)^2}{2} - \frac{\psi^2}{2}\right) + nD = \\ &= R\left(\frac{\varphi^2}{2} - \frac{\psi^2}{2}\right) + \frac{H}{n}\left(\frac{\varphi^2}{2} - \frac{\psi^2}{2}\right) + l\left(\frac{(\varphi - 2\beta)^2}{2} - \frac{\psi^2}{2}\right) + nD. \end{aligned} \tag{6}$$

When deriving formula (6), it was meant that the geometric path of the ray incident on the semitransparent mirror and reflected from it is equal to D. From equality (6), if we take into account the constancy of the value $(\varphi + \psi)$, we obtain that the path difference rays is also determined by a linear dependence on the angle φ or, respectively, on the angle ψ. Consequently, the step of alternating interference fringes in a rough approximation is equal to a constant value.

Expanding the differences of squares (6) and using (3) we get:

$$\Delta S = \frac{D}{2}(\varphi - \psi) + \frac{2l(\beta^2 - \varphi\beta)}{(R + H/n + l)} + \frac{-D\varphi + 2n\beta l}{2}(\varphi - \psi) + nD. \tag{7}$$

Neglecting the second and third terms in equality (7) due to their smallness, ΔS can be represented as:

$$\Delta S = \frac{D}{2}(\varphi - \psi) + nD. \tag{8}$$

To find the width of the interference fringe, we change the angles φ and ψ so that the variation in the optical path difference is equal to the wavelength. From equalities (4) and (8), we obtain that the magnitudes of changes in angles satisfy the conditions

$$\begin{cases} \Delta\varphi + \Delta\psi = 0, \\ \frac{D}{2}(\Delta\varphi - \Delta\psi) = \lambda \end{cases}$$

or $\Delta\varphi = -\Delta\psi = \lambda / D.$

Then from formula (1) it follows that the value of the width of the interference fringe h is equal to

$$h = \frac{D+l}{D}\lambda. \tag{9}$$

The negative sign (omitted in formula (9)) says that the direction of alternation of the bands is opposite to the direction of the *Ox* axis. From formula (9) it follows that in order to increase the measurement accuracy, it is necessary to choose the base *D* as small as possible, and the distance to the analysis line as large as possible within the limits admissible by the design constraints and the conditions for attenuating the distortions of the interference pattern caused by factors of a higher order of smallness.

The phase difference corresponding to the stroke difference is:

$$\delta = \frac{2\pi}{\lambda}\Delta S.$$

We denote by m the nearest integer to $\Delta S / \lambda$ not exceeding it. The quantity

$$e = \Delta S/\lambda - m,\ 0 \le e < 1$$

called the fractional order of interference. This value determines what is observed at a given point of the analysis plane - a minimum or a maximum (if e = 0, then at this point a maximum illumination is observed, if $e = 0.5$ is a minimum of illumination).

The angular displacement of the radiation source in the analysis plane, provided there is no radial displacement, leads to a change in the angle ψ by $\Delta\psi$ or to an equal, opposite in sign, change in the angle φ by $\Delta\varphi$. This will lead to a shift of the interference band by an amount

$$\Delta h = -(D+l)\,\Delta\varphi.$$

Accordingly, the magnitude of the displacement of the strip is the magnitude of the angular displacement of the source

$$\Delta\varphi = -\frac{\Delta h}{(D+l)}. \tag{10}$$

Let us substitute in formula (10) the expression obtained from (9)

$$\Delta\varphi = -\frac{\Delta h}{Dh}\lambda = -\frac{\Delta h}{h}\frac{\lambda}{D}. \tag{11}$$

Formula (11) is the basis of the mathematical model for analyzing the interference pattern and is the main one for determining the angular displacement, provided that there is no radial displacement and displacement perpendicular to the analysis plane. To determine the displacement perpendicular to the plane of analysis, a second interferometer is required with a datum perpendicular to the plane of the section. If the distance to the radiation source is known, then at small angles the transverse displacement is found by the formula:

$$s = R\Delta\varphi.$$

It can be seen from formula (4) that the radial displacement of the radiation source, provided that there is no tangential displacement, leads to a change in $\Delta(\psi + \varphi)$ equal to $\Delta(\psi+\varphi) = -\dfrac{D}{(R+H/n+l)^2}\Delta R.$

(12)

RESULT AND DISCUSSION

The analysis of radial displacement requires taking into account values of a higher order of smallness. If we assume approximately that the quantities *1/R* and ΔR are quantities of the same order of smallness, then the radial displacement affects the interference pattern in quantities of the third order. The above analysis in lower approximations for solving the problem of radial displacement requires significant refinement. But since the problem of determining the angular displacement is of the greatest interest, the influence of the radial displacement at the first stage can be neglected. Therefore, we restrict ourselves to the results of the analysis.

The analysis line can also be positioned in a plane perpendicular to the plane viewed. This will lead to the need to additionally take into account the passage of rays through the part of the base adjacent to the right mirror. In this case, the qualitative picture will not change. The quantitative change is reduced, basically, to the appearance in the denominator of formula (4) of an additional term corresponding to the path of rays through this part of the base [4].

CONCLUSION

Note that to analyze the displacement, it is necessary to have two frames that fix the interference region for two points in time that determine the displacement. Since the region of interference has a periodicity, then that part of the displacement is revealed, which is expressed by the fractional part of the value Δh. Therefore, the quantization of motion should be such that for a quantum of time the value of the displacement Δh satisfies the inequality:

$$-0{,}5\,h < \Delta h < 0{,}5h$$

which is sufficient for the accuracy of determining the displacement value. In the future, this condition is assumed to be met.

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